PROPERTIES OF SOLUTIONS
Chapter 13
13.4 Expressing Solution Concentration
13.5 Colligative Properties
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## Molarity (M)

Molarity: number of moles of solute per liter of Solution.
$\mathbf{M}=\frac{\text { moles solute }}{\mathrm{L} \text { solution }} \equiv \frac{\mathrm{mol}}{\mathrm{L}} \equiv \frac{\mathrm{mmol}}{\mathrm{mL}}$
Number of mol = M x V (L)
Number of $\mathbf{m o l}=\mathbf{M x V}(\mathrm{mL})$
Properties: T-dependent.

## Concentration Units

## Solution:

homogeneous mixture of two or more components

Solute + Solvent $\rightarrow$ Solution
Concentration of solute:
either Amount of solute/amount of solvent
or Amount of solutelamount solution
Amount = moles, mass, or volume

Molality: number of moles of solute per kg of solvent.
$m=\frac{\text { moles solute }}{\text { kg solvent }} \quad M=\frac{\text { moles solute }}{\text { Lsolution }}$
Number of mol $=\mathbf{m x k g}$ solvent
Properties: T-independent.

## Molality (m)

$$
\begin{aligned}
& \text { Mole Fraction ( } X \text { ) } \\
& X=\frac{\text { mol component }}{\text { total mol }} \\
& X_{\mathrm{i}}=\frac{n_{\mathrm{i}}}{n_{\mathrm{t}}} \\
& \text { Component } \mathrm{A}=2 \mathrm{~mol} \\
& X_{\mathrm{A}}=\frac{2}{5}=0.4 \\
& X_{1}+X_{2}+X_{3}+\ldots .=1 \\
& \text { Component } \mathrm{B}=3 \mathrm{~mol} \\
& X_{\mathrm{B}}=\frac{3}{5}=0.6 \\
& \text { T-independent }
\end{aligned}
$$

## Mass Percent

percent by mass $=\frac{\text { mass component }}{\text { mass solution }} \times 100 \%$
$36 \% \mathrm{HCl}$ solution by mass:

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100 g solution \equiv 36 g HCI + 64 g H2O
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mass $_{1} \%+$ mass $_{2} \%+\ldots=100$
T-independent

A sulfuric acid solution containing 571.6 g of $\mathrm{H}_{2} \mathrm{SO}_{4}$ per liter of solution has a density of $1.329 \mathrm{~g} / \mathrm{mL}$. Calculate (a) the mass percent; (b) the mole fraction; (c) the molality; (d) the molarity of $\mathrm{H}_{2} \mathrm{SO}_{4}$ in this solution.
mass $\%=\frac{\text { mass solute }}{\text { mass solution }} \times 100 \%$
mass solt $=571.6 \mathrm{~g}$
mass soln $=1000 . \mathrm{mL} \times \frac{1.329 \mathrm{~g}}{\mathrm{~mL}}=1329 \mathrm{~g}$
mass $\%=\frac{571.6 g}{1329 g} \times 100 \%=43.01 \%$

$$
\begin{aligned}
& \text { mass } \mathrm{H}_{\mathbf{2}} \mathrm{O}=\text { mass of soln }- \text { mass of solt } \\
& \text { mass } \mathrm{H}_{\mathbf{2}} \mathrm{O}=1329-571.6=757.4 \mathrm{~g} \mathrm{H} \mathbf{2} \\
& n_{H_{2} O}=757.4 \mathrm{gH}_{2} \mathrm{O} \times \frac{1 \mathrm{~mol} \mathrm{H}}{\mathbf{2}} \mathbf{O} \\
& n_{H_{2} \mathrm{O}}=42.02 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} \\
& X_{H_{2} \mathrm{SO}_{4}}=\frac{5.833}{5.833+42.03}=0.1219 \\
& X_{H_{2} \mathrm{O}}=1-X_{H_{2} S O_{4}}=1-0.1219=0.8781
\end{aligned}
$$

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## Colligative Properties


$\mathrm{m}=\frac{\text { moles solute }}{\mathrm{kg} \text { solvent }}$
$\mathrm{m}=\frac{5.833 \mathrm{~mol} \mathrm{H}_{2} \mathrm{SO}_{4}}{0.7574 \mathrm{~kg} \mathrm{H}_{2} \mathrm{O}}=7.701 \mathrm{~m}$
$M=\frac{\text { moles solute }}{L \text { solution }}$
$M=\frac{5.833 \mathrm{~mol}}{1 \mathrm{~L}}=5.833 \mathrm{M}$
$\Theta$
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## Colligative Properties

Vapor Pressure Lowering
Boiling Point Elevation

Freezing Point Depression
Osmotic Pressure

## Colligative Properties

Properties that depend on the concentration (number) of solute particles in solution, not on the nature of the solute particles.
As the concentration of solute particles increases:

The vapor pressure
The boiling point
The freezing point
The osmotic pressure
decreases increases decreases increases

## Vapor Pressure Lowering

- Adding a nonvolatile solute to a solvent always lowers the vapor pressure of the solvent.

$P^{\circ}$ solv

$\boldsymbol{P}_{\text {soln }}$


## Colligative Properties

| Conc. | \# of particles | $\Delta \mathrm{T}_{\mathrm{b}}$ |
| :--- | :---: | :--- |
| 1 m glucose | 1 | $1.5^{\circ} \mathrm{C}$ |
| 1 m urea | 1 | $1.5^{\circ} \mathrm{C}$ |
| 2 m glucose | 1 | $3^{\circ} \mathrm{C}$ |
| 1 m NaCl | 2 | $3^{\circ} \mathrm{C}$ |
|  |  |  |
|  |  |  |

## Raoult's Law

The partial pressure of a solution is directly proportional to the mole fraction of the solvent in that solution.
$P_{1}=X_{1} P_{1}^{\circ}$
$P^{\circ}{ }_{1}=$ vapor pressure of pure solvent
$X_{1}=$ mole fraction of the solvent


## Raoult's Law

$P_{1}=X_{1} P_{1}^{o}$
If solution contains only one solute:
$X_{1}=1-X_{2} \quad X_{2}=$ mole fraction of solute
$P_{1}^{\circ}-P_{1}=\Delta P=X_{2} P^{\circ}{ }_{1}$
The lowering in vapor pressure, $\Delta P$, is directly proportional to the solute concentration, $X_{2}$.

What is the vapor pressure of a solution made by mixing 150.0 g glucose
$\left(\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}\right)$ with 600.0 g water at $25^{\circ} \mathrm{C}$ ? (at $25^{\circ} \mathrm{C}$ the vapor pressure of pure water is 23.8 torr.)
$P_{1}=X_{1} P_{1}^{\circ}$

$$
\begin{aligned}
& 150.0 \mathrm{~g} \text { glu } \times \frac{1 \mathrm{~mol}}{180.2 \mathrm{~g} \mathrm{glu}}=0.8324 \mathrm{~mol} \\
& 600.0 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} \times \frac{1 \mathrm{~mol}}{18.02 \mathrm{~g} \mathrm{H}_{2} \mathrm{O}}=33.30 \mathrm{~mol}
\end{aligned}
$$

For two volatile liquids:
$P_{\mathrm{t}}=P_{\mathrm{A}}+P_{\mathrm{B}}$
$P_{t}=X_{A} P_{A}^{\circ}+X_{B} P_{B}^{o}$
$X_{\mathrm{A}}$ : mole fraction of liquid A in liquid phase
$X_{B}$ : mole fraction of liquid B in liquid phase

$$
\begin{aligned}
& X_{H_{2} O}=\frac{n_{H_{2} O}}{n_{H_{2} O}+n_{C_{6} H_{12} O_{6}}} \\
& X_{H_{2} O}=\frac{33.30}{33.30+0.8324}=0.9756 \\
& P_{1}=X_{1} P_{1}^{\circ}{ }_{1} \\
& P_{1}=0.9756 \times 23.8=\mathbf{2 3 . 2} \mathbf{~ m m H g}
\end{aligned}
$$


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## Boiling Point Elevation

$K_{\mathrm{b}}=\frac{\Delta T_{\mathrm{b}}}{m}=\frac{{ }^{\circ} \mathrm{C}}{m}$
For water $K_{\mathrm{b}}=\mathbf{0 . 5 2}{ }^{\circ} \mathrm{C} / \mathrm{m}$
Each mole of solute particles raises the boiling point of 1 kilogram of water by $0.52{ }^{\circ} \mathrm{C}$.
$K_{\mathrm{b}}$ : characteristic of solvent

## Boiling Point Elevation

$\Delta T_{b}=K_{b} \cdot m$
$\Delta T_{\mathrm{b}}=T_{\mathrm{b}}-T_{\mathrm{b}}{ }^{\circ}$
$T_{\mathrm{b}}=$ boiling point of solution
$T_{\mathrm{b}}{ }^{\circ}=$ boiling point of pure solvent
$K_{b}=$ molal boiling point elevation constant
$m=$ molality of the solution

## Freezing Point Depression

$\Delta T_{f}=K_{f} . m$
$\Delta T_{f}=T_{f}^{\circ}-T_{f}$
$T_{f}=$ freezing point of solution
$T_{f}^{\circ}=$ freezing point of pure solvent
$K_{f}=$ molal freezing point depression constant
$\boldsymbol{m}=$ molality of the solution

## Freezing Point Depression

$K_{\mathrm{f}}=\frac{\Delta \boldsymbol{T}_{\mathrm{f}}}{\boldsymbol{m}}=\frac{{ }^{\circ} \mathrm{C}}{\boldsymbol{m}}$
For water $K_{f}=1.86{ }^{\circ} \mathrm{C} / m$
Each mole of solute particles lowers the freezing point of 1 kilogram of water by $1.86^{\circ} \mathrm{C}$.
$K_{\mathrm{f}}:$ characteristic of solvent

List the following aqueous solutions in order of: (a) increasing freezing point; (b) increasing boiling point:

|  | concentration of particles |  |
| :--- | :---: | :--- |
| $\left(\mathrm{NO}_{3}\right)_{2}$ | $1 \times 3$ | $=3 \mathrm{~m}$ |
| 3 m KCl | $3 \times 2$ | $=6 \mathrm{~m}$ |
| 4 m C | $\mathrm{H}_{12} \mathrm{O}_{6}$ | $4 \times 1$ |

What is the freezing point of a solution containing 478 g of ethylene glycol $\left(\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}_{2}\right.$, antifreeze) in 3202 g of water?

Molal Boiling Point Elevation and Freezing Point Depression Constants

| Solvent | $K_{\mathrm{b}}$ | $K_{\mathrm{f}}\left({ }^{\circ} \mathrm{C} / m\right)$ |
| :--- | :--- | :--- |
| Water | 0.52 | 1.86 |
| Acetic Acid | 3.7 | 3.90 |
| Benzene | 2.53 | 5.12 |
| Phenol | 3.56 | 7.27 |
| Carbon | 4.02 | 29.8 |
| Tetrachloride |  |  |

$$
\Delta T_{f}=K_{f} . m \quad K_{f}=1.86^{\circ} \mathrm{C} / m
$$

$$
m=\frac{\mathrm{mol} \mathrm{solt}}{\mathrm{~kg} \mathrm{solv}}=\frac{478 \mathrm{~g} \mathrm{x} \frac{1 \mathrm{~mol}}{62.01 \mathrm{~g}}}{3.202 \mathrm{~kg}}=2.41 \mathrm{~m}
$$

$m=\frac{\text { mol solt }}{\mathrm{kg} \mathrm{solv}}=\frac{478 \mathrm{~g} \mathrm{x} \frac{1 \mathrm{~mol}}{62.01 \mathrm{~g}}}{3.202 \mathrm{~kg}}=2.41 \mathrm{~m}$
(a) $\mathrm{KCl}<\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}<\mathrm{Ca}\left(\mathrm{NO}_{3}\right)_{2}<\mathrm{H}_{2} \mathrm{O}$
(b) $\mathrm{H}_{2} \mathrm{O}<\mathrm{Ca}\left(\mathrm{NO}_{3}\right)_{2}<\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}<\mathrm{KCl}$

$$
\begin{aligned}
& \Delta T_{f}=K_{f} \cdot m \quad K_{f}=1.86{ }^{\circ} \mathrm{C} / m \\
& \Delta T_{f}=\frac{1.86{ }^{\circ} \mathrm{C}}{m} \times 2.41 \mathrm{~m}=4.48^{\circ} \mathrm{C} \\
& \Delta T_{f}=T_{f}^{\circ}-T_{f} \\
& T_{f}=T_{f}^{\circ}-\Delta T_{f} \\
& T_{f}=0.00^{\circ} \mathrm{C}-4.48^{\circ} \mathrm{C}=-4.48^{\circ} \mathrm{C}
\end{aligned}
$$

What volume of ethyleneglycol $\left(\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}_{2}\right)$ must be added to 15.0 L of water to produce an antifreeze solution with a freezing point of $-30.0^{\circ} \mathrm{C}$ ? What is the boiling point of this solution? Density of $\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}_{2}=1.11 \mathrm{~g} / \mathrm{mL}$ and for water $1.00 \mathrm{~g} / \mathrm{mL}$.
$\Delta T_{f}=T_{f}^{\circ}-T_{f}=0.00^{\circ} \mathrm{C}-\left(-30.0^{\circ} \mathrm{C}\right)=30.0^{\circ} \mathrm{C}$
$\Delta T_{f}=K_{f} . m$
$30.0=1.86{ }^{\circ} \mathrm{C} / \mathrm{m} \times \mathrm{m}$
$m=16.13 \mathrm{~m}=16.13 \frac{\mathrm{~mol} \mathrm{eg}}{\mathrm{Kg} \mathrm{H}_{2} \mathrm{O}}$
$\Theta$
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## Osmotic Pressure

Osmosis:
Flow of solvent molecules through a semipermeable membrane from a dilute solution to a more concentrated one
semipermeable membrane:
Allows the passage of solvent molecules but not the solute molecules


## Osmotic Pressure:

The minimum pressure that stops osmosis.

| $\pi=M R T$ | $\pi=$ osmotic pressure in atm |
| :--- | :--- |
|  | $M=$ molarity of solution |
|  | $R=$ gas constant |
|  | $T=$ temperature in K |
|  | isotonic solutions |
| $\pi_{1}=\pi_{2}$ | soln $_{1}:$ hypertonic concentrated |
| $\pi_{1}>\pi_{2}$ | soln $_{2}:$ hypotonic diluted |
|  | 34 |

Determination of Molar Mass
Any of the colligative properties can be used for molar mass determination of the solute with osmotic pressure being the most accurate.
$\Leftrightarrow$ solution 35

A 20.0 mg sample of a protein is dissolved in 25.0 mL of solution. The osmotic pressure of the solution is 0.56 torr at $25^{\circ} \mathrm{C}$. What is the molar mass of the protein?
$M=\frac{\pi}{R T}=\frac{0.56 \text { torr } \times \frac{1 \mathrm{~atm}}{760 \mathrm{torr}}}{0.0821 \frac{\mathrm{~L} . \mathrm{atm}}{\mathrm{mol.K}} \times 298 \mathrm{~K}}$
$M=3.01 \times 10^{-5} \mathrm{M}$
$\Theta$

When 0.186 g of an organic substance is dissolved in 22.01 g of liquid camphor $\left(\mathrm{C}_{10} \mathrm{H}_{16} \mathrm{O}\right)$, the freezing point of the solution is found to be $176.7^{\circ} \mathrm{C}$. What is the molar mass of the organic compound?
For camphor: $T^{\circ}{ }_{f}=179.8^{\circ} \mathrm{C}, K_{f}=40.0^{\circ} \mathrm{C} / \mathrm{m}$
$\Delta T_{f}=K_{f} m$
$\Delta T_{f}=T_{f}^{\circ}-T_{f}=179.8-176.7=3.1^{\circ} \mathrm{C}$
$3.1^{\circ} \mathrm{C}=40.0^{\circ} \mathrm{C} / \mathrm{m} \times \mathrm{m}$
$m=0.0775 m$
$3.01 \times 10^{-5} \frac{\mathrm{~mol}}{\mathrm{~L}} \times \frac{25.0 \mathrm{~mL} \times 1 \mathrm{~L}}{1000 \mathrm{~mL}}$

$$
=7.5 \times 10^{-7} \mathrm{~mol}
$$

$\mathscr{M}=\frac{20.0 \mathrm{mg} \times \frac{1 \mathrm{~g}}{1000 \mathrm{mg}}}{7.5 \times 10^{-7} \mathrm{~mol}}=2.67 \times 10^{4} \mathrm{~g} / \mathrm{mol}$
$\frac{0.0775 \mathrm{~mol} \text { solt }}{\mathrm{kg} \text { solv }} \times 0.02201 \mathrm{~kg}$ solv $=1.71 \times 10^{-3} \mathrm{~mol}$
$\mathcal{M}=\frac{\mathrm{g}}{\mathrm{mol}}=\frac{0.186 \mathrm{~g}}{1.71 \times 10^{-3} \mathrm{~mol}}=109 \mathrm{~g} / \mathrm{mol}$

