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Q.A.J.U.S.T

BIOSTATISTICS

لطلبة الصيدلة والعلوم الطبية

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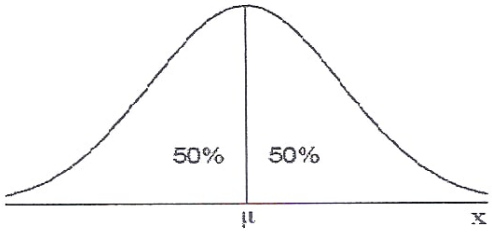
The Normal Distribution

It is defined as a continuous frequency distribution of infinite range. (can take any values not just integers)
It is important tool in analysis of epidemiological data and management science.

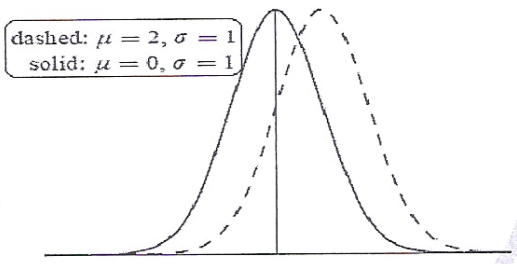
Characteristics of normal distributions:

(1) Bell shaped.
(2) Suitable for continuous variables.
(3) Symmetry about the mean, μ
(4) Mean = median = mode.
(5) The total area bounded by its curve and the x-axis is equal to 1.
(6) 50% of the area is to the left of the mean, and 50% to the right.

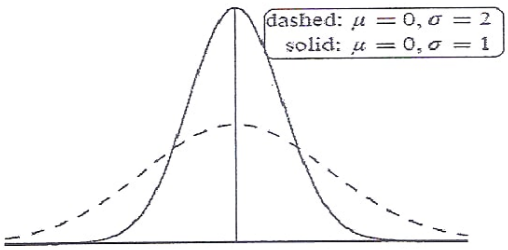


(7) Theoretically, curve extends to infinity.

(8) Depends on two parameters: (a) the mean μ, (b) the standard deviation σ.
Different values of μ shift the curve along the x-axis (Fig. a) so it affects the location of the curve.
Different values of σ determine the flatness (spread) of the curve (Fig. b) so it determines the curve shape.



(Fig. a)



(Fig. b)

(9) The area under the curve represent probability or proportion.

One way to find areas under the curve is using tables:

Use standard normal curve. It has $\mu = 0$, and $\sigma = 1$
The standard normal curve has the same shape of normal distribution curve.
Transform the variable (x) into a continuous random variable Z using the following formula:

$$z = \frac{\text{Value - Mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma}$$

So, **Z score**: is the distance between a selected value, designated (x), and the population mean, divided by the population standard deviation.

Z score is calculated to 2 decimal places.

After calculating Z score use Z table (see last page) to find the areas (probabilities) to the left of Z value as follows:

How to find Z value from the table?

Example: To find the area to the left of the value of $Z = 0.24$, look for 0.2 in the vertical column and for 0.04 in the horizontal one then take the value of their intersection as shown below:

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0										
0.1										
0.2					0.5948					

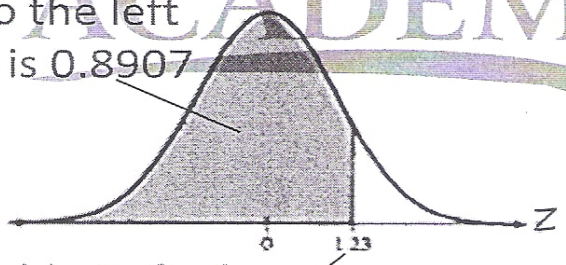
Then the area to the left of Z 0.24 is 59.48%

Remember: At Z = 0 the area to the left is 0.50

Finding Probability (area) under the curve using Z tables

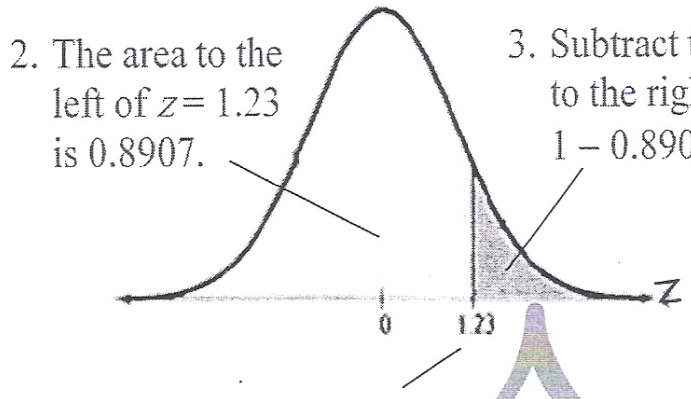
Case 1: To find the area to the *left* of *Z*, find the area that corresponds to *Z* direct in the Standard Normal Table: (see last pages)

2. The area to the left of $z = 1.23$ is 0.8907

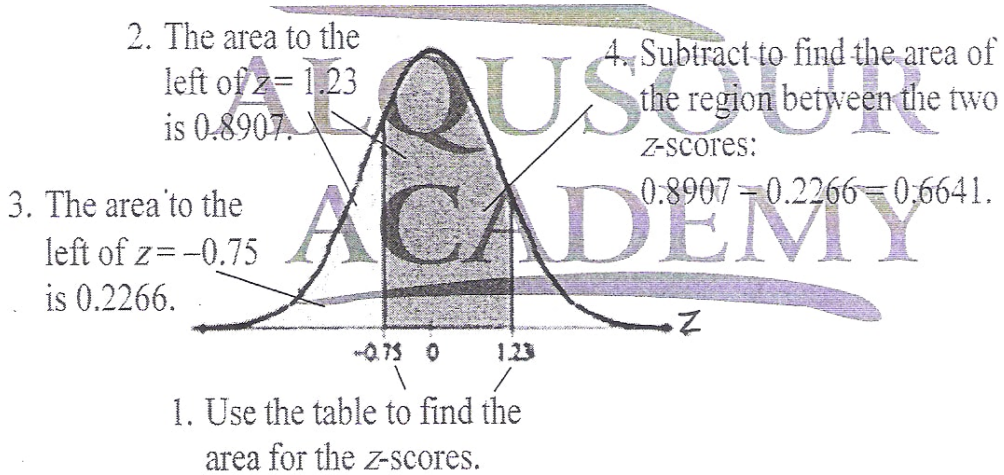


1. Use the table to find the area for the *z*-score

Case 2: To find the area to the *right* of *Z*, use the Standard Normal Table to find the area that corresponds to *Z*. Then subtract the area from 1.



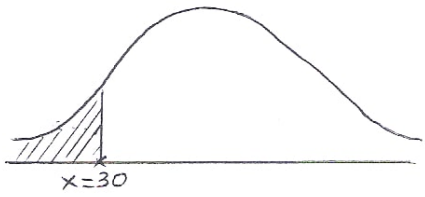
Case 3: To find the area *between* two *Z*-scores, find the area corresponding to each *Z*-score in the Standard Normal Table. Then subtract the smaller area from the larger area.



Example

Suppose the average length of stay in a hospital is 60 days with standard deviation of 15. Assuming normal distribution of lengths of stay:

(a) **Find the Probability that a randomly selected patient from this group will have a length of stay Less than 30 days.**



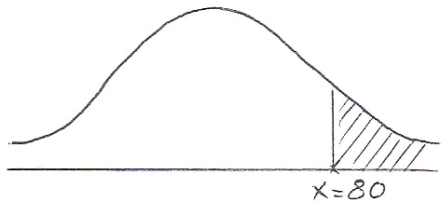
Solution:

Transform $x = 30$ in to Z using the following formula:

$$Z = \frac{30-60}{15} = -2$$

then find the area to the left of $Z = -2$ direct from the table. It is 0.0228

(b) **Find the Probability that a randomly selected patient from this group will have a length of stay Greater than 80 days.**



Solution:

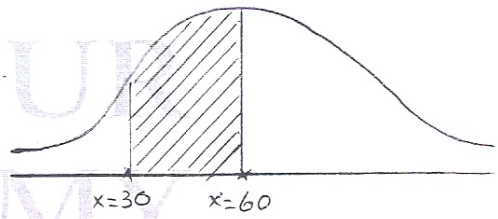
Transform $x = 80$ in to Z using the following formula:

$$Z = \frac{80 - 60}{15} = 1.33$$

Then the area at 1.33 (area to the left) is 0.9082 from the table. We want area greater than 80, then Area:

$$= 1 - P(Z \leq 1.33) = 1 - 0.9082 = 0.0918$$

(c) **Find the Probability that a randomly selected patient from this group will have a length of stay Between 30 and 60 days.**



Solution:

Transform $x = 60$ in to Z using the following formula:

$$Z = \frac{60 - 60}{15} = 0$$

Transform $x = 30$ in to Z using the following formula:

$$Z = \frac{30 - 60}{15} = -2$$

The probability at $Z = 0$ from the table is 0.5

The probability at $Z = -2$ from the table is 0.0228

Then subtract the largest area from the smallest one as follows:

$$= P(Z \leq 0) - P(Z \leq -2) = 0.5 - 0.0228 = 0.4772$$

Example

Find the Z-score that corresponds to an area of 0.9050 under the standard normal curve to the left of the Z-score.

Solution:

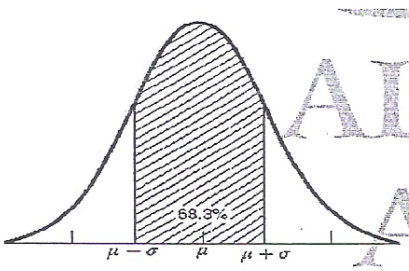
Look at the area 0.9050 in the middle of Z-table. The close value to it is 0.9049, then the Z-value for it is 1.31.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9090	.9115	.9131	.9147	.9162	.9177

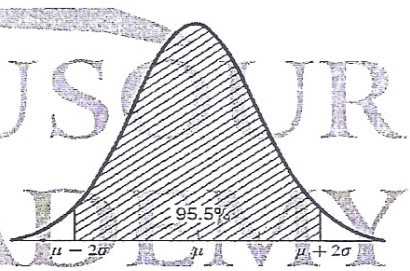
Another method to find probabilities is using empirical rule:

- 68% of the area lie within ±1 standard deviation from the mean.
- 95% of the area lie within ±2 standard deviations from the mean.
- 99.7% of the area lie within ±3 standard deviations from the mean .

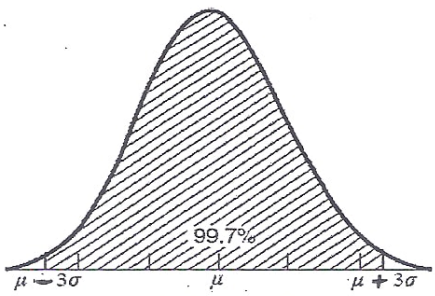
So almost all of the distributions will lie within 3 deviations of the mean



(Fig. a)



(Fig. b)

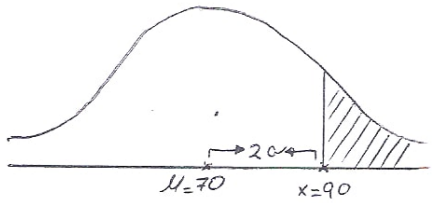


(Fig. c)

Range	Area in the range
$-1\,\sigma, +1\,\sigma$	68 %
$-2\,\sigma, +2\,\sigma$	95 %
$-3\,\sigma, +3\,\sigma$	99.7 %
Between Q_{25} and Q_{75}	50%

Exercise:

If heart beats is normally distributed with mean 70 beats/min and standard deviation 10. Answer questions 1 to 3?



1) What area of the curve is above 90 beats/min?

Answer: here, $Z = \frac{90-70}{10} = 2$

Direct to the Z table the area to the left of Z is 0.9772. So, the area to the right is:

$$1 - 0.9772 = 0.023$$

Then the area above 90 beats/min is 2.3%

Or without using tables, 90 is 2 standard deviation away from the mean, the area between 50 and 90 is 95%. Then the area out of this range is 5%.
We want the area in the right tail so it is $5\% \div 2 = 2.5\%$

2) What area of the curve is between 40 and 100 beats/min?

Answer:

Both 40 and 100 are 3 standard deviation away from the mean, then the area between them is 99.7%.

3)What area of the curve is below 40 beats per min or above 100 beats per min?

Answer: 0.3% because here we need both tails outside the 99.7% area.

Example

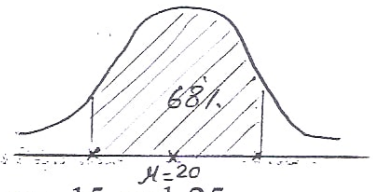
The daily water usage per person in Irbid is normally distributed with a mean of 20 gallons and a standard deviation of 5 gallons.

Answer questions 1 to 3:

1) About 68% of the daily water usage per person lies between what two values?

Solution:

68% is between $\pm 1\sigma$.



Then the value to the left of the mean is: $\mu - 1\sigma = 20 - 5 = 15$.

Then the value to the right of the mean is: $\mu + 1\sigma = 20 + 5 = 25$.

That means about 68% of the daily water usage will be between 15 and 25 gallons.

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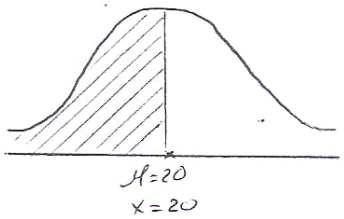
2) What is the probability that a person from new area selected at random will use less than 20 gallons per day?

Solution:



Z score for 20 is:

$$Z = \frac{20 - 20}{5} = 0$$

Then the area to the left (less than Z = 0) is 0.5 (50%).

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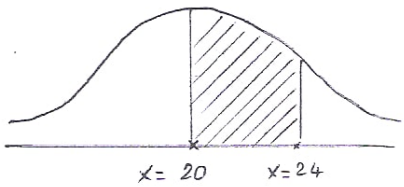
3) what percent uses between 20 and 24 gallons?

Solution:

- The Z score for $x = 20$ is $Z = 0$

and the area to the left of 20 is 0.5



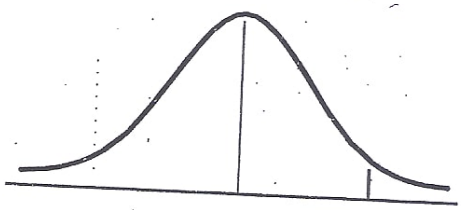
- The Z score for $x = 24$ is:

$$Z = \frac{24 - 20}{5} = 0.8$$

and the area to the left of 24 from the table is: 0.78814

- To find the area between 20 and 24 substract their areas:
  $0.78814 - 0.5 = 0.2881$ that is 28.8%

- Z indicates how many standard deviations away from the mean the point (x) lies.



Comparing x and Z units

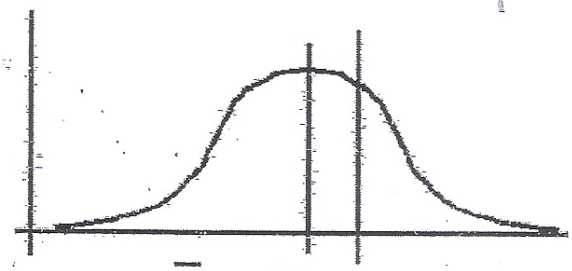
Why to use Z-scores?

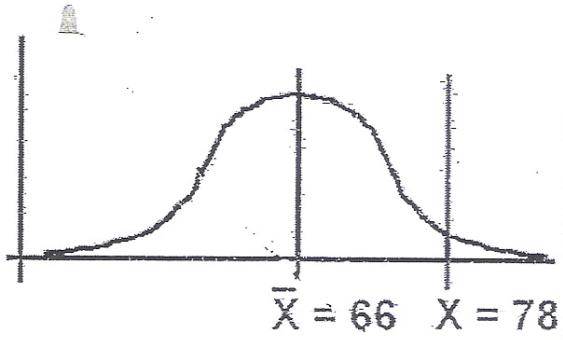
Z-scores make it easier to compare scores from distributions using different scales.

Example

On test A, Fred scores 78. Mean score = 70, SD = 8.
On test B, Fred scores 78. Mean score = 66, SD = 6.
Did Fred do better or worse on the second test?

Solution:
Test A: as a Z-score, Z = (78 –70) /8 = 1
Test B: as a Z-score, Z = (78 – 66) /6 = 2
So, Fred did much better on test B.





A Z- value of 2 indicates that the value of 78 is 2 standard deviation above the mean 66.

Uses of normal distribution:

1) In statistical inferences.
2) In hypothesis testing.(Z score used with t-score, chi square and F-statistics)
3) Help managers to make decisions.

Example

Birth weights at a certain hospital are normally distributed with mean = 112 oz and standard deviation = 21 oz.

1) What is the z-score for an infant with birth weight =154 oz.?

$$Z = \frac{x - \mu}{\sigma} = \frac{154 - 112}{21} = 2$$

2) How many standard deviations above the mean is this birth weight?

2 standard deviation. (this indicates that the value 154 oz is 2 standard deviation above the mean 112 oz).

3) How many standard deviations below the mean is a birth weight of 91 oz?

$Z = \frac{x-\mu}{\sigma} = \frac{91-112}{21} = -1$ (this indicates that the value 91 oz is 1 standard deviation below the mean 112 oz).

Note: Not all continues variables are normally distributed.

Are my data normally distributed?

- It should be bell shaped.(histogram can show bell shape)
- Mean = median = mode.
- If 68% of data lie within 1 standard deviation, 95% of data lie within 2 standard deviation of the mean, or 99.7% of data lies within 3 standard deviation of the mean.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997