ALQUSOUR ACADEMY
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Q.A.J.U.S.T

BIOSTATISTICS

لطلبة الصيدلة والعلوم الطبية

Subject:

Final Exam – Part One

ALQUSOUR ACADEMY

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HYPOTHESIS TESTING

Inferential statistics:

It is using sample data to draw conclusions about populations.

Before generalizing any research results we should remember that <u>Results may be:</u>

1) True. (there is really relation or difference).
2) due to chance (we know it by hypothesis testing).

<u>The role of hypothesis testing:</u>

To tell whether the association is true or due to chance. (It finds the probability that the observed association is due to chance). So we do testing to remove the effect of chance.

<u>To make a hypothesis and test it we have to:</u>

1) Understand the nature of the data (i.e. discrete vs. continuous; # and type of variables) which determines the particular test to be employed.
2) Verify Assumptions: general procedure is modified depending on the assumptions (i.e. normality, equality of variances, independent samples).

A **hypothesis** is a statement about one or more populations.

We have two types:

1) The **null hypothesis**, denoted by H_0, is the hypothesis to be tested. It is the hypothesis of no difference or no association. It is **opposite our believe.**
(the difference is due to random chance).

Example: There is no association between air conditioning and allergy.

2) The **alternative hypothesis**, denoted by H_A or H_1, is a statement of what we will **believe** is true . (conclusion the researcher is trying to reach).
(the difference is real).

Usually, the alternative hypothesis and the research hypothesis are the same.

Example: Air conditioning causes allergy.

How to write null and alternative hypothesis:

- The null and alternative hypothesis are complementary.
- In writing null and alternative hypothesis use population parameters like μ not sample estimates like $\bar{x}$
- Equality appears in H_0.
- We compare a sample statistic to a population parameter to see if there is a significant difference.
- In the "one sample" case, we compare with population parameter (in the next example it is 10).

Example on writing H_0 and H_A:

Suppose we want to test that the mean time to deliver pizza is 10 minutes, then the null and alternative hypothesis will be:

H_A: $\mu \neq 10$ (there is difference)

H_0: $\mu = 10$ (there is no difference)

Suppose we believe that the mean time to deliver pizza is less than 10 minutes, then the null and alternative hypothesis will be:

H_A: $\mu < 10$

H_0: $\mu \geq 10$

Important Example:

If H_A: Boys are smarter than girls, then the null hypothesis will be:

a) There is no difference in cleaverness between boys and girls.

b) Girls are smarter than boys.

The answer is b.

Types of Errors:

1) Type I Error: A Type I error occurs when we reject a true null hypothesis. The probability of committing a Type I error is called α or significance level.

2) Type II Error: A Type II error occurs when we fail to reject (accept) a false null hypothesis.

Decision	H_0 True	H_0 False
Fail to reject H_0 (accept)	Correct decision	**Type II Error** (Beta)
Reject H_0	**Type I Error** (α)	Correct decision (test power)

Power: It is rejecting a false null hypothesis.

Power $= 1 - \beta$

Type I error is more serious than Type II error.
It is more serious to send an innocent man to jail, than set a guilty man free.
Thus, we are more concerned with controlling Type I error than Type II error.

As you decrease α, you increase β and vice versa.
Thus, general strategy: fix α and try to minimize β.

Hypothesis could be one tailed or two tailed. This depends on what we want to test:

Types of Tests			
Test	H_0	H_A	**Rejection region**
Two-tailed Test	$=$	$\neq$	Fig. 1
Left-tailed Test	$\geq$	$<$	Fig. 2
Right-tailed Test	$\leq$	$>$	Fig. 3

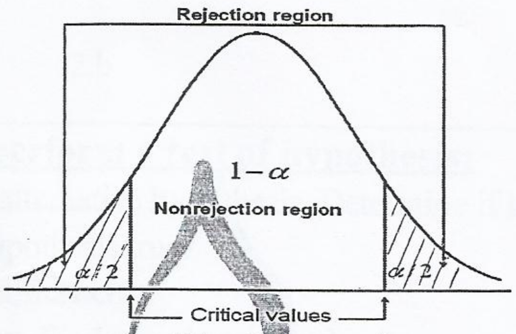


Fig. 1

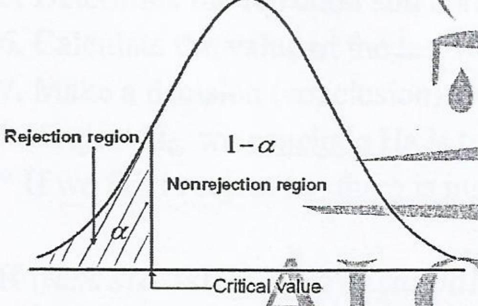


Fig. 2

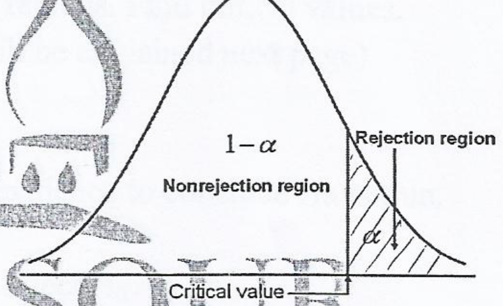


Fig.3

Note: critical values found from tables.

Some Common Critical Values:

α	**0.1**	**0.05**	**0.01**
$Z_{(\alpha)}$ when area to left (Fig. 2)	−1.28	−1.645	−2.33
$Z_{(1-\alpha)}$ when area to right (Fig. 3)	1.28	1.645	2.33
$Z_{(1-\frac{\alpha}{2})}$ when having 2 tails (Fig.1)	1.645	1.96	2.575

- The values of the rejection region are those values that are less likely to occur if the null hypothesis is true.

Exercises:

1) In hypotheses testing, a type I error occurs when:

a. a true null hypothesis is rejected. **b.** false null hypothesis is rejected

c. a false null hypothesis is not rejected **d.** none of the above

2) When we accept a false null hypothesis, then we:

a. commit a type I error **b.** commit a type II error

c. do a correct decision **d.** none of the above

Answers : 1.a 2.b

General steps to perform a test of hypothesis:

1. Write the null and alternative hypothesis. Determine if it is one or two tailed.
2. Assume the null hypothesis true.
3. Set the level of significance.
4. Collect evidence (ex. Find sample average).
5. Determine the rejection and non-rejection regions. Find critical values.
6. Calculate the value of the test statistic. (will be explained next page)
7. Make a decision (conclusion):

* If reject H_0, we conclude Ha is true.
* If we fail to reject H_0, there is insufficient evidence to conclude Ha is true.

If $|test\ statistic| \geq |critical\ values|$ then reject H_0 .
Or if the test statistic falls in rejection areas then reject the null.

Hypothesis Testing: A Single Population Mean:

CASE 1:

Assumptions: Sampling is Random from a normally distributed population with known variance and large sample size ($n \geq 100$):

In this case use Z (normal) distribution.

Alternative Hypothesis	Critical values (tabulated values)
$H_A : \mu \neq \mu_0$ (two tailed) $H_A : \mu < \mu_0$ (one tailed) $H_A : \mu > \mu_0$ (one tailed)	$-Z_{(1-\frac{\alpha}{2})}$ or $Z_{(1-\frac{\alpha}{2})}$ $-Z_{(1-\alpha)}$ $Z_{(1-\alpha)}$

μ_0 is called a **reference value** or **standard value.** It is given in the question (population value). In writing H_0 and H_A, we compare with it.

Test statistic (standardized value):

$$\frac{\textit{point estimator} - \textit{hypothesis parameter}}{\textit{standard error}} =$$

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} \quad : \sim N(0,1)$$

- We used Z because σ is known.
- **Note:** $Z \sim N(0,1)$ this means it is normally distributed with mean zero and varience 1.
- Test statistic is also called calculated value.

Important Note: In the exam, if α not given in the question, consider it 0.05

Example:

The mean serum creatinine level measured in 12 patients 24 hours after they received a newly proposed antibiotic was 1.2 mg/dl. Suppose serum creatinine level in the general population is normally distributed with mean and standard deviation 1.0 and 0.4 mg/dl, respectively.

Can we conclude that the mean serum creatinine level in this group is more than that of the general population.

(a) State the null and alternative hypotheses.

Solution: $H_0 : \mu \leq 1.0$ and $H_A : \mu > 1.0$.

(b) Using a significance level of 0.05, find the critical value of the test.

Solution: This is a Right-tailed test. (confidence level = 0.95), using the z table $Z_{(1-\alpha)} = 1.65$.

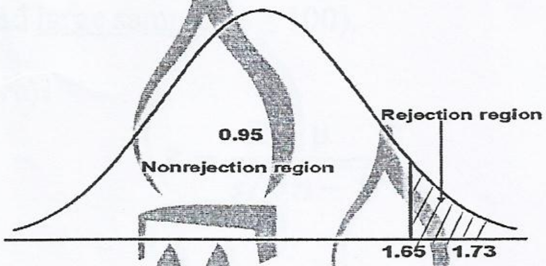
(c) Find the value of the test statistic.

Solution:

$$Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{1.2-1}{0.4/\sqrt{12}} = 1.732$$

(d) What is your conclusion?

Solution: We shall reject H_0 since $|1.732| > |1.65|$. (i.e. H_A is true, that is, the mean serum creatinine level in this group is greater than that of the general population).



- Increasing the level of significance (α) will increase the rejection region, and vise versa.

-When we decrease the confidence level ($1-\alpha$), then we increase the level of significance (α). So decreasing the confidence level will increase the rejection region, and vise versa.

Exercise:

1) If a hypothesis is rejected at 99% confidence, then it:

a. will always be rejected at 95% confidence **c.** a + b

b. will always be rejected at 90% confidence **d.** none of these

Answer: 1. c

PRECAUTION:
Hypothesis Testing does not lead to proof of hypothesis, it merely indicates whether the hypothesis is supported or not supported by the available data.
When we fail to reject Ho, we do not say the Ho is true, but that there is insufficient evidence to conclude the Ha. May be underpowered (this is related to sample size n).

CASE 2:

Assumptions: Sampling is random from a normally distributed population with unknown variance and large sample ($n \geq 100$).

Test statistic (Z score):

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

Example:
A nonsmoker male believes that smokers differ in sleeping hours than the general public which needs an average of 7.7 hours of sleep each day. He questioned 151 smokers about the number of hours they sleep each day, the sample mean was 7.5 hours and the standard deviation was 0.5 hours. Suppose you are asked to help this person about his belief.

(a) Write out the null and alternative hypothesis for your test.

Solution:
- $H_0 : \mu_s = 7.7$

(or H_0: no difference between the sample mean and population parameter, then $\bar{x} = \mu$). So, the difference is due to chance.
In other wards: the sample of 151 comes from a population that has a mean of 7.7.

- $H_A : \mu_s \neq 7.7$.

(or H_A: there is a difference between the sample mean and the population parameter). So, the difference is real.
Or in other wards the sample of 151 comes from a population that does not have a mean of 7.7. In reality, it comes from a different population.

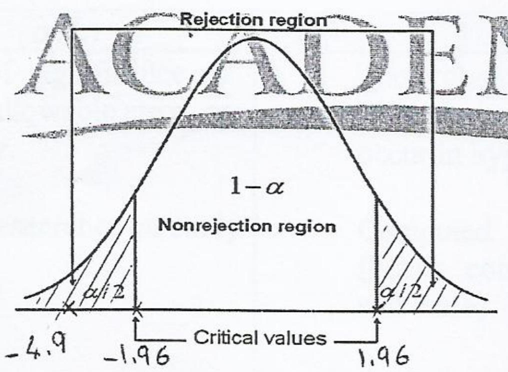
(b) If you let the confidence level 95%, what is the critical values?
Solution: This is a two-tailed test. $1 - \alpha = 0.95$. So the critical values :
$Z = \pm 1.96$.

(c) What is the value of the test statistic?
Solution: Test statistic is:

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{7.5 - 7.7}{0.5/\sqrt{151-1}} = -4.90$$

(d) What is your statistical conclusion?
Solution: Reject H_0, since $-4.90 < -1.96$.(the difference is significant, so it is unlikely to have occurred by random)

(e) What you should tell this person about his belief?
Solution: His belief is true, that is, the smokers differ in sleeping hours than the general which needs on average about 7.7 hours.



Note:

α is the indicator of rare event.

Any difference with a probability less than α is rare and will cause us to reject the null hypothesis.

Significant does not mean a relation is important and it is not necessary causal one. It means there is relation only.

In decision making, to decide if there is a significant result:

It is important to report not just that we found a difference, but how big it is, and the precision of our estimate. That's why *P*- value is important.

***P*-value**: the probability of finding the observed finding or difference is due to chance.

The p-value is the probability calculated under the Ho (assume H_o is true) that the test statistic takes on a value equal to or more extreme than the value observed.

The p-value indicates how strong (or weak) the rejection of the H_o will be.

➔ Small p-values signify strong rejection of H_o

It must be noted, that no p-value, however small, excludes chance completely.

The difference between *α* and *p*-value :

α	*P*-value
• It is Level of significance or Maximum allowable error or Type 1 error. • Set by the researcher (at study beginning).	• The probability of chance • The exact value of error that occur in hypothesis testing. • Computed from the sample (better computed by special soft ware) • At the end of study.

- If ***P*-value** $< \alpha$ then we **reject null** (there is significant different), it is not likely due to chance. That means the chance has a very minimal effect on this difference so it is a significant difference: (real difference or association)

- If ***P*-value** $\geq \alpha$ then we **accept null.**

So, if we reject null hypothesis, this means P-value should be $< \alpha$.

How to calculate P-value?

Example on two tails:

The mean serum creatinine level measured in 12 random sample of patients after they received a newly proposed antibiotic was 1.2 mg/dl. Suppose serum creatinine level in the general population is normally distributed with mean 1.0 and standard deviation (σ) 0.4 mg/dl.

Suppose we are interested in testing whether the mean serum creatinine level in this group is different from general population mean of 1.0.

Solution:

n = 12

$\bar{x} = 1.2$

$\mu = 1$

$\sigma = 0.4$

$H_0 : \mu = 1.0$

$H_A : \mu \neq 1.0$

Solution:

Since the $\bar{x} = 1.2$ is to the right of the mean (μ=1), Calculate Probability of ($\bar{x} \geq 1.2$)

To find the probability convert $\bar{x}$ *to* $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{1.2-1}{0.4/\sqrt{12}} = \mathbf{1.73}$

then find the probability $P(Z \geq 1.73) = 1 - 0.9582$ *(from table)* $= 0.042$

So, P-value = $2 \times 0.042 = 0.084$. (we have two tailed in the question so we multiplied the answer by 2)
P-value (0.084) > α (0.05), then we accept the null hypothesis.

So, the mean serum creatinine level in the sample is not different from that of the general population.

Summary of methods of hypothesis testing:

1) Calculate test statistic value as we did in this part.

2) From p-value :
- If **P-value < α** then we **reject null** (the probability of getting result due to chance is close to zero).
- If **P-value ≥ α** then we **accept null.**

3) If zero included in confidence interval, then we fail to reject null (H_0) hypothesis, and vise versa.
